Computational Models of Decision Making

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# Introduction

## Overview

There have been many descriptions of the competing forces that shape human and animal decision. The first, referred to as “model free,” involves behaviors that are instinctual or habitual. The second, known as “model based,” encompasses behaviors that are planned or thought out. This paper will begin by giving a brief historical overview of the experiments and research that gave rise to the modern field of reinforcement learning. It will then more closely examine the distinction differing implications of model free and model based learning, including a general overview of both types of learning, as well as a more detailed account of how computational models would be created based on each paradigm. These computational models will be used to address the question of which type of decision making is dominant in human use. Analyses will be conducted to determine whether parameterizations of these models can be used to predict differences in subjects’ reaction times, based on predictions made from computational theory on the relation between computational complexity and processing time. Finally, recent research in the field will be discussed in the context of these models.

## History and Origins

The algorithms, techniques, and methods associated with reinforcement learning in modern computational neuroscience stemmed from what were, at the time, considered two very different fields. The first was the study of animal intelligence. Many of the original reinforcement learning techniques and experimental paradigms were developed based on simple trial and error experiments in the field of animal learning. In one of the first scientific texts on the subject, Edward Thorndike discussed his observations of instinctual and habitual animal behavior, paving the way for what would become known in the field as model free learning (Thorndike, 1898).

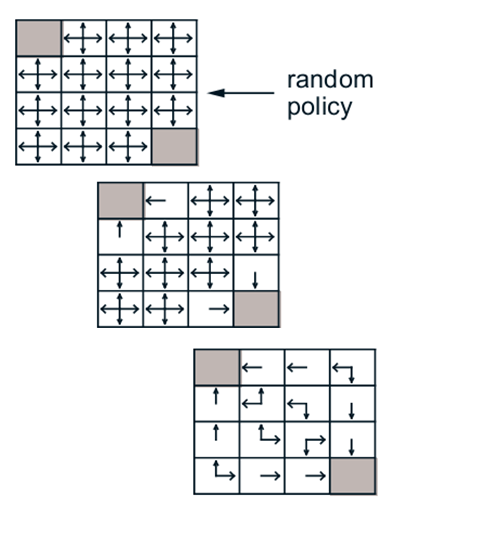
Half a century later, the same concepts were addressed from a completely different perspective, with a much more abstract approach. Many of the most basic and fundamental equations we use in reinforcement learning stem from this time period. Richard Bellman, a prominent computer scientist at the time, laid the groundwork for the field in the 1950’s, after he was inspired by Claude Shannon’s earlier suggestion that “a computer could be programmed to use an evaluation function to play chess” (Sutton and Barto, 1998). Bellman used numeric representations of the current state and available actions, coupled with a value function (or “optimal return function”), to help the computational agent evaluate which action was most likely to result in maximal reward. The new field, christened “dynamic programming” quickly converged with earlier work in animal psychology, giving researchers a more rigorous framework with which to make and assess predictions about animal behavior and the underlying neural circuitry.

The first rigorous computational investigations of the phenomenon of trial and error learning were conducted simultaneously in 1954, by both Minsky and by Farley and Clark. The term “reinforcement learning” was used for the first time just eleven years later, in Waltz and Fu’s 1965 “A heuristic approach to reinforcement learning control systems”. Importantly, certain characteristics from both fields remained. Reinforcement learning must be both selectional and associative. Selectional learning, as Sutton and Barto, put it, “involves trying alternatives and selecting among them by comparing their consequences,” while associative refers to a process where “the alternatives found by selection are associated with particular situations.” (Sutton and Barto, 1998). Selection is closely related to the process of searching, while association is more closely related to remembering. As an example of the boundaries of each, consider the phenomena of natural selection and supervised learning. Natural selection exemplifies selective processes, but is not associative, while supervised learning embodies associative learning, but is not selective. (Sutton and Barto, 1998)

Research continued rapidly through the end of the previous century and the beginning of the current one. Sutton, in a series of papers published in 1978, brought the fields of dynamic programming and animal psychology closer together than ever by developing a set of learning rules to link changes in predictions of the same quantity made at successive points in time (Sutton 1978), connecting current computational models to past studies done in animal behavior. Major progress in the field was made again by Sutton in 1988, after the use of temporal difference (TD) learning as a general prediction method (Sutton, 1988). The power of reinforcement learning, and more specifically TD learning was highlighted by the success of Tesauro’s 1992 “TD-Gammon,” a “a neural network that trains itself to be an evaluation function for the game of backgammon by playing against itself and learning from the outcome”, (Tesauro, 1992) bringing a new wave of attention to the field and hearkening in the modern era of reinforcement learning.

## Implementation of Reinforcement Learning

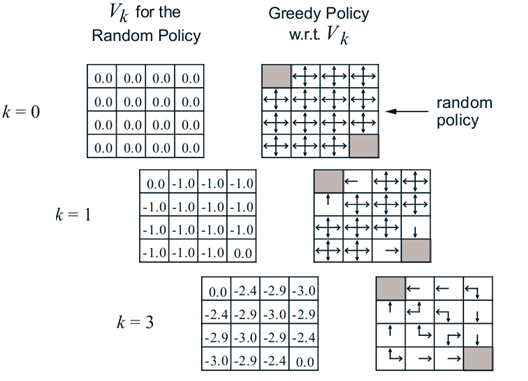
There are many different ways to implement reinforcement learning, but all share some basic features and vocabulary. First and foremost, all implementations of reinforcement learning involve the decisions and resultant actions of a computational entity, known as the “agent”. Importantly, the tenets of reinforcement learning described above still apply: the agent is not told which actions to take, but instead must make choices that display a balance between a greedy exploitation of the information it currently knows and an adventurous exploration of the information it’s unaware of. Importantly, the agent must traverse the environment without a teaching signal other than the rewards it receives. Reinforcement learning is not an instance of supervised learning, so the agent must determine how best to optimize rewards by itself



*Figure 1.1: A Policy*

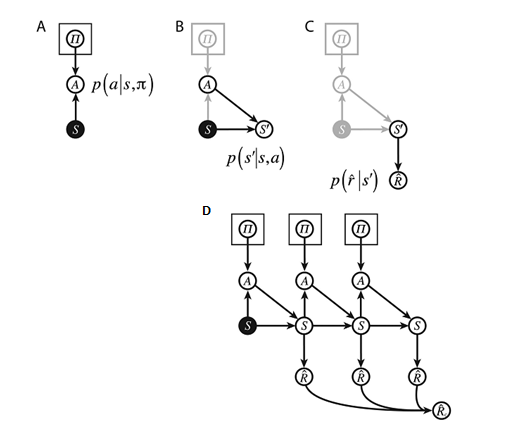
In this example, the agent’s task was to navigate from one of the un-shaded starting cells to either of the two shaded corner cells, without selecting a move that would result in moving off the end of the grid. The state is determined by the current cell the agent is in. Each cell is representative of a different state. Each cell shares the same available set of actions: moving up, down, left, or right. The policy, which in this example is stochastic, is represented by the arrows within the cells. Note how the policy changes over time, moving away from actions which would move the agent off the edge and tending towards actions which would lead towards the rewarded shaded cells. Adapted from Sutton and Barto, 1998

The agent is able to do so by using a mapping of discrete situations, known as “states” to actions to take in those states. This pairing of states and optimal actions is known as a policy (see Figure 1.1). The agent updates its policy after every new action is taken based on both its reward function and its value function. The reward function maps each state to the expected reward to be received upon entering that state. It is essential to reinforcement learning, but is inherently myopic; the reward function can only consider immediate rewards, and cannot look ahead to see what rewards might result from later actions beginning from the current state. The value function corrects the short sightedness of the reward function by specifying what’s good in the long run (see Figure 1.2). The value of a state is representational of the total amount of reward an agent can expect to accumulate over the future, beginning from the current state. Finally, the model’s policy, current state, and chosen action can be linked together in what’s known as a state-action diagram (see Figure 1.3 for further explanation). (Sutton and Barto, 1998)



*Figure 1.2: A Value Function*

The agent must again navigate the same scenario described in Figure 1. The corresponding value function, or the agent’s representation of the expected reward from each state, is shown in the left hand grid. The value function is computed by the agent, based on both the transition and reward functions. In this example, the agent receives a reward of -1 for any action in any state. In any state, choosing an action causes the agent to deterministically move one cell in that direction, except those actions which would cause the agent to move off the grid. In those cases, the state of the agent remains unchanged, although the agent still receives a reward of -1 for the action. The values of -3.0 in the center cells reflect that the agent expects it will take three moves to reach the goal state from the current state. The higher than expected values of -2.4 in the cells adjacent to the goal state reflect the random nature of the policy. Even though the goal state may be reached with one action, it may take more than one choice of action before the correct action is selected stochastically. Finally, the value of 0.0 for the goal state reflects that the agent expects it will take zero moves to reach the goal state (i.e. it has already reached the goal state). Adapted from Sutton and Barto, 1998



*Figure 1.3: A State-Action Diagram*

**Figure A**: Given a state *s* and a policy *π* with which to evaluate it, the agent can select an action *a*.

**Figure B**: Given a state *s* and action *a*, the agent finds itself in a new state *s’.*

**Figure C**: After finding itself in a new state *s’*, the agent receives a reward .

**Figure D**: This process can be iterated over, in each case feeding the new state *s’* generated by the selection of action *a* under policy π to come to a new state *s*’’, resulting in a new reward . These rewards are summed over all the iterations to produce reward *R.* For all processes described in Figures A-D, the underlying selection mechanics can be either stochastic or deterministic Adapted from Solway and Botvinick, 2012

## Model Free and Model Based Learning

Importantly, some agents include what’s known as a “model”. The model is an internal representation that mimics the behavior of the environment. When given both a state and an action, the model would attempt predict the resultant state and the next reward. Models are therefore the basis of any planning done by the agent. Two straightforward distinctions arise from the introduction of models to reinforcement learning. Agents that contain models are termed “model-based”, while agents without models are referred to as “model-free”. While conceptually straight forward, analogues of these processes in humans are thought to play a large role in the neural processes underlying addiction and substance abuse (Everitt and Robbins, 2004)

We will give a quick example to illustrate the differences between model free and model based learning. First, let us consider the model based case. As described above, model based agents have an internal model of the environment, which they can tweak or manipulate to determine the best possible action. Let us consider a driver (we’ll call him Paul) who, upon entering a four way intersection, encounters a large pothole across his normal path straight through the intersection. Paul brings up a mental picture of his normal route, and realizes that if he makes a right, two quick lefts, and another right, he will be back on his normal route, beyond the obstruction. Paul’s use of an internal representation of the environment (his mental map) makes this a prime example of a model-based decision.

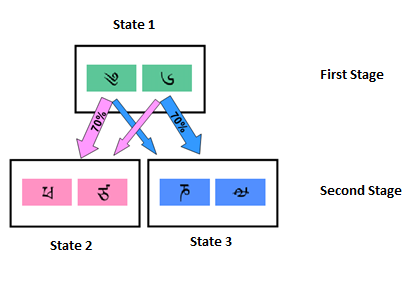
A few moments later, Bill arrives at the same intersection, along his (identical) route to work. Bill, however, makes his decisions in a completely model free way. In past instances where his normal path at this intersection was blocked by huge potholes (this is an especially unlucky intersection), things worked out well when Bill made a right turn. Bill has no idea what will happen when he goes to the right, or what actions he will take once he does, but he is not worried about planning ahead. Bill’s lack of plan or internal map of his route, coupled with complete reliance on instinct, help him exemplify the model free decision making process.

# Nathaniel Daw’s Study

## Overview

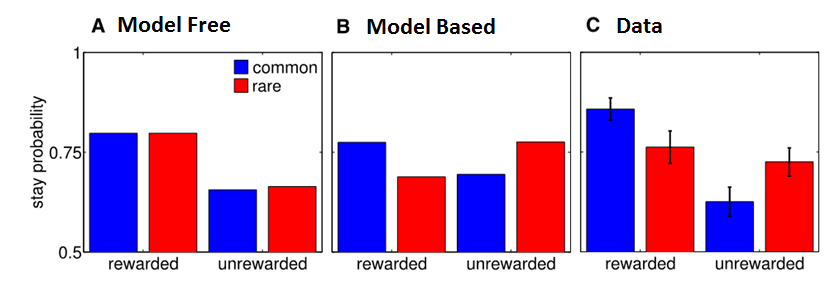
Nathaniel Daw recently conducted a study (Daw et al, 2011) to investigate the relative contributions of model free and model based agents in human decision making. He collected data from 17 participants through 201 trials of a two stage decision task. On each trial, subjects were presented with a choice of two options, each of which was labeled with a semantically irrelevant Tibetan character. Choice of either of these two options (we’ll refer to them as left and right, although throughout the study the same characters appeared on both the left and right sides), lead probabilistically two one of two second stage states, which we’ll refer to as states 2 and 3. The transition probabilities were arranged so that choosing left in the first (initial) stage lead to state 2 70% of the time (the “common” transition) and state 3 30% of the time (the “rare” transition), while the reverse was true for choosing right (see Figure 2.1). These transition probabilities were fixed throughout the course of the experiment.

Once the initial choice had been made, subjects were presented with an additional choice between two new options, each labeled again with Tibetan characters. The two possible second stage states (state 2 and state 3) were distinguished both by the background color of the choice and by the characters labeling the choices. The subjects were rewarded probabilistically after choosing an option in the second stage. To encourage continued learning throughout the trials, the chance of receiving a reward after choosing a given second stage action was updated after each trial by a random Gaussian walk.

  
*Figure 2.1: Daw’s Two Stage Decision Task*

Each trial consisted of two stages. In the first stage, subjects were presented with two choices, labeled by semantically irrelevant Tibetan characters. Choice of the left character (in this depiction) led to State 2 on seventy percent on trials (the “common” transition) and to State 3 on thirty percent of trials (the “rare” transition). The opposite held true for an initial choice of the right character. In the second stage, subjects entered into one of two States (State 2 or State 3), depending probabilistically on their first stage choice. The probability of a reward for each of the four possible second stage choices varied, and was updated after each trial by a random Gaussian walk. (Daw et al., 2011)

At this point, we can begin to see the elegance of the task in its ability to tease out the relative contributions of model free and model based decision processes. The key is to look at the subject’s behavior on trials which were ultimately rewarded, where the first stage choice followed the “rare” transition path to the second stage state. On the next trial, a purely model-free agent would display an increased tendency to make the same first stage choice, because that first stage choice was ultimately rewarded on the last trial. However, a model-based agent would make the opposite first stage choice. Although this may at first seem counter-intuitive, upon reexamination it makes sense. The model-based agent is aware of the transition probabilities associated with each of the first states (indeed, that is the entirety of the “model”). In order to have the best chance of entering the same second stage state it was last rewarded in, it should choose what it knows is the “common” transition path to this state, as opposed to the “rare” transition path it took on the previous trial. The expected stay probabilities on successive first stage choices for model free and model based agents, along with the actual findings from the study, can be seen in Figure 2.2. Special attention is due to Figure C, the report of the patient data from the study, which suggests that human decision making is a mixture of both model free and model based processes.

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*Figure 2.2: Expected and Actual Stay Probabilities*

**Figure A:** Model free agents would be expected to make the same first stage choice after a trial that was rewarded, regardless of whether it entered the second stage via the common or the rare transition path.

**Figure B:** Model based agents would be expected to take advantage of their knowledge of the environment. If a trial was rewarded after entering the second stage via a common transition path, the agent would be expected to make the same first stage choice again. In contrast, if a trial was rewarded after entering the second stage via a rare transition path, it would be expected to make the opposite first stage choice, in order to maximize the chance of reaching the same second stage state. The same holds true in reverse for unrewarded trials

**Figure C:** The actual data. Note the results were a mix of both the model free and the model based expectations, suggesting human decision making is a mixture of both processes. (Daw et al., 2011)

## Construction of Model Free and Model Based Computational Agents

It’s easy to see that Daw’s experimental paradigm lends itself well to the construction of model free and model based agents. First and foremost, the number of possible states and decisions from those states is finite. There are three possible states (one first stage state and two second stage states) and two possible actions from each of those states (left or right) for a total of six possible state action pairs. The value function can therefore be represented as a 2x3 matrix, with one cell for each possible state-action pair.

To better understand the contributions of model free and model based agents to the human decision making, we constructed a total of three separate models – a model free model, a model based model, and a hybrid of the two – which were run in parallel throughout the simulation (see appendix for full code base). Importantly, the hybrid model allowed the relative strength of model free and model based processes to be quantified numerically, through the use of the parameter *w*, which will be described further below. The overall parameterization of the models including *w*) allowed any correlations between the parameters and the subjects’ reaction time to be revealed.

We will begin by discussing the model free model. Learning at the first level was conducted using the SARSA equation (Sutton and Barto, 1998). In the SARSA equation, the model’s estimate of the values of the first stage choices are updated after the second choice is made. The agent calculates the difference between its estimate of the value of the second choice and its estimate for the first choice. It then multiplies this difference by a factor *eta*, known as the learning rate, and adds the product to its estimate of the chosen action in the first state. A similar calculation is performed to determine how to update the value for the second state, except in this calculation the difference is found between the expected value of the second stage choice and the value of the reward received after the choice. It is again multiplied by a factor *eta*, although the *etas* for each stage are allowed to vary independently, to better capture differences in learning rates between stages. Finally, the first level values are updated again to reflect the change in estimated values at the second stage. However, this secondary round of first level updates is multiplied by an additional factor *lambda*, known as the eligibility trace, which allows the model to discount more temporally distant influences.

Next, the model based agent is constructed. The agent begins by examining all the previous transitions from the first stage in previous trials to make an educated guess about which first level choice is most frequently associated with each of the two second stage states. It does so using a for loop, examining all trials 1:i-1 (from the first trial to the trial before the current one) and counts the number of time each possible assignment of transition probabilities (i.e. whether choosing left in stage one lead to state two or state three more frequently, indicating which is the “common” transition path). It then updates the value estimates for each of the first stage choices. The agent does so by doing a weighted sum of the maximum of the estimated values for the second and third states. For example, if the agent had decided that choosing left in the first state lead to the second state seventy percent of the time, it would update its estimate of the value of choosing left in the first state as follows:

*(0.70)(highest valued choice in second state) +*

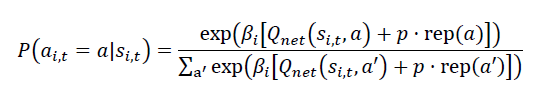
*(0.30)(highest valued choice in third state)*

The value of choosing right in the first state would be updated in a similar fashion, with the position of the 0.70 and 0.30 weights reversed. Note the absence of lambda or eta from the first stage of learning in model based agents. The transition probabilities are implicit in the model, so there’s no need to guess at the values of first stage choices, and therefore no need for a factor of *eta*. Similar logic explains the absence of the *lambda* parameter.

Learning in model based agents in the second level is conducted identically to the second level for model free agents. This makes sense, as reward after a second stage choice is assigned probabilistically according to a random Gaussian walk, so there are no transition probabilities to model. Finally, we consider the construction of a hybrid model of the two. Once the other agents have been created, formation of a hybrid is trivial. The values of the hybrid model, for both the first and second stage, is updated on each trial as a weighted sum of the model free and model based agents, governed by a free parameter *w:*

*Qhybrid = w \* Qbased + (1-w) \* Qfree;*

The probability of each choice in the model was calculated using the softmax:



*Beta* represents the inverse temperature, and governs how deterministic the model’s choices are. Similar to *eta, beta* is allowed to vary between stages to capture any differences between first and second stage choices. Rep(a) is an indicator function that evaluates to one when the first stage choice was the same on this trial as it was on the previous trial, and evaluates to zero otherwise. The free parameter *p* acts as a weighting mechanism for the indicator function, allowing the model to account for first-order perseveration (*p > 0)* and switching (*p < 0).*

Overall, there were seven free parameters used in the model (*eta* at the first and second stages, *beta* at the first and second stages, *lambda, w, and p*). Models were constructed for each of the seventeen subjects using the methods described above. One subject’s data was removed after an analysis of their model, which displayed an extremely fast reaction time, indicting the subject was not fully engaged in the task. For each subject, trials where the reaction time was +/- two standard deviations from their mean reaction time were ignored. The best fitting models were found by maximizing the log likelihood of the fit (in practice, minimizing the negative log likelihood) using MATLAB’s function fmincon(). We created models for each subject a total of ten times, from a random set of initial parameters, to ensure fmincon() had not settled in a local minimum.

# Expansion of Daw’s Research

## Overview

After construction of the models, we conducted a post hoc analysis of data provided to us by Nathaniel Daw, which was collected during his 2011 study. We assessed the computed parameters’ ability to predict reaction times and searched for correlations between any of the variables. Importantly, these correlations were unexamined by Daw and his colleagues.

## Motivation

Computational theory predicts that model based learning would be more computationally demanding than model free learning. This makes sense, and has close analogies to processing and memory access time in computer science. In general, model free decisions require no taxing computations, while model based ones can be incredibly computationally demanding. This is evident both from the code base and from the underlying theory. Think back to the state-action diagram portrayed in Figure 1.3. As it is on the page, the diagram only shows the selected action in each state. Consider what it would look like if the diagram contained all of the possible actions from each state, instead of only the selected one. Now consider if each of those actions lead to a new state, and new potential actions radiated from each new state. In complex scenarios, the agent must traverse each of these potential routes and select the path that yields the greatest total reward, in a manner similar to a depth-first search. Searches like as these can be incredibly time consuming and computationally demanding.

Though Daw’s experimental paradigm is constructed to minimize the number of possible ways to traverse the decision task, we can see an analogue to the demanding computations described above present in our code (see appendix). The model based agent requires an additional calculation to be performed using the Bellman equation. In this case, as there are only a maximum two states to look back upon, the calculation is trivial. However, the computational complexity would grow exponentially with each new level added. Even in this trivial form, it is still more computationally demanding than its analogue in the model free agent, which requires no additional computation at the moment of decision. If the neural relationship between computational complexity and processing time is analogous to the silicone one, subjects who were model based in their decision making would be expected to have slower reaction times than subjects who were model free.

The above reasoning can be understood in a more concrete way as well. Consider the example of Paul at the intersection mentioned earlier. His decision was quick, and based solely on habit and instinct. Specifically, in the computational agents, the model free case requires no strenuous computations on its behalf. Instead, it only needs to access its previous estimates of values, and update the values based on prediction error. This situation case is closely analogous to accessing pre-cached value on a computer’s memory chip, where the processor only needs to read a value from memory, but doesn’t need to do any lengthy calculations with it. Based on this logic, one would expect people whose decisions were largely model-free to have faster reaction times.

Following similar logic in reverse, we would expect model based calculations to take longer. Think back to Bill’s behavior when confronted with an obstacle at the intersection described above. Bill conjured a mental picture of his route, and then used basic route planning methods to find a way around the obstacle. This takes significantly more time. Bill must consider all of the possible options, and pick a new route that will work, and hopefully be the fastest. With each new intersection encountered, the number of possible pathways that Bill must consider grows exponentially.

## 3.3 Supporting Literature

There’s some data to support this view in the current literature as well. Otto and his colleagues recently conducted a study investigating the effect of computational load on the relative contributions of model free and model based decision making agents. To do so, they used a task that was functionally equivalent to Daw’s two stage decision tree described above (using fractals in place of Tibetan characters), with the inclusion of an additional hundred trials. On one hundred pseudo-randomly selected of these trials, participants were required to simultaneously complete a Numerical Stroop task. Importantly, the subjects were instructed to focus primarily on the working memory task, and to make choices with “what was left over”.

To analyze their results, Otto sorted each subject’s trials into three groups, based upon when the most recent Stroop trial had occurred. The trials were denoted Lag-0, Lag-1, and Lag-2, and referred to cases where the Stroop trial occurred on the current trial, the previous trial, or the trial before the previous trial, respectively. They found a distinct difference in decision making strategies used on Lag-0 trials when compared to Lag-2 trials. On trials with current computational load (Lag-0), subjects did not use their knowledge of the transition structure of the task, while on normal trials (Lag-1 and Lag-2), subjects used a mixture of model free and model based decision processes, similar to both previous data (Daw, 2011) and our results described above. Otto’s findings imply computational load has an impact on the way subjects make their decisions.

Otto’s findings are encouraging for our hypothesis as well. First, they further validate Daw’s decision task and experimental paradigm, identical to the set up our study used. More importantly, the results demonstrate a link between the available cognitive resources and the proportion of model free and model based behavior, implying model based decisions are more computationally demanding than model free ones. This link is critical; it makes intuitive sense that more complex trials would require more time to process, increasing the subject’s expected reaction time

# Results

We ran a series of post hoc statistical analyses to assess any correlations between the parameters of the model and reaction time on each of the trials. To conduct the analyses, we used the R statistics software package, including the ltm (Latent Trait Models) library, as well as the Matlab programming environment. We began by conducting a percentile analysis of the model parameters and log likelihoods of the fits across subjects, which can be seen in Table 4.1. Importantly, the observed distributions were consistent with previous analyses, including Daw’s own (Daw et al, 2011).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Eta 1st | Eta 2nd | Beta 1st | Beta 2nd | Lambda | W | P | Log Likelihood |
| 25th percentile | 0.4483 | 0.2118 | 3.298 | 2.6934 | 0.4154 | 0.094 | 0.0524 | 167.8667 |
| 50th percentile | 0.5813 | 0.42 | 5.1938 | 3.6936 | 0.5595 | 0.3704 | 0.1413 | 197.2618 |
| 75th percentile | 0.9059 | 0.7083 | 7.4349 | 5.1018 | 0.9198 | 0.5221 | 0.2127 | 227.5732 |

*Table 4.1: Percentile Analysis of Parameter Distribution*

We began by assessing the correlation between *w* and average reaction times at the first level. If model based learning was slower than model free learning, there should be a correlation between the subjects’ relative levels of model free and model based decision making and how quickly they responded. No significant correlation or trend was found (see Appendix A, Table 1). Next, to better standardize first level reaction times across subjects, a multiple regression analysis was conducted with the inclusion of all parameters in the model (see Appendix A, Table 2). No significant correlation was found between *w* and first level reaction times even after the inclusion of all other parameters, although a positive trend between *eta* and the second stage and first level reaction times was revealed. The above analysis was repeated, allowing for interactions between *w* and all parameters uniquely used in model free learning, with similar results (see Appendix A, Table 3).

Further multiple regression analyses were conducted between various subsets of the model parameters and first stage reaction time (not included). In almost all cases, the observed relationship between second level *eta* and first level reaction times was confirmed. All regressions conducted displayed a positive relationship between first level RTs and *w*, though few were significant. One regression was especially of interest, due to the observed trend between *w* and first stage RT’s, and is included below.

We examined the combined ability of *w, eta* at the first level, *eta* at the second level, and *beta* at the second level to predict the subject’s average reaction time at the first level:

*(r1 ~ w + eta1 + eta2 + beta2)*

The overall correlation between the first level reaction times and the above parameters was found to be significant, with p < 0.05. The *eta* parameters at both the first and second stages were found to be the most significant contributors to the regression, with p < 0.01 and p < 0.05, respectively. Importantly, there was a weakly significant positive correlation between *w* and first level reaction times, with p < 0.10. For a full summary of the regression analysis, see Table 4.3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Estimate** | **Std. Error** | **t value** | **Pr(>|t|)** | **Significance** |
| **(Intercept)** | 583.328 | 59.109 | 9.869 | 8.44E-07 | \*\*\* |
| **w** | 123.931 | 60.2 | 2.059 | 0.06402 | x |
| **Eta 1st** | -135.071 | 57.283 | -2.358 | 0.03795 | \* |
| **Eta 2nd** | 185.656 | 57.789 | 3.213 | 0.00827 | \*\* |
| **Beta 2nd** | 16.121 | 8.103 | 1.99 | 0.07208 | x |

Significance Codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘x’ 0.1 ‘ ’ 1

*Table 4.3: Multiple Regression Analysis*

Finally, pairwise correlation analyses were conducted for all model parameters, as well as first and second level reaction times. The tests revealed a few interesting relationships. There was a significant negative correlation observed between the first level inverse temperature parameter *beta* and the decay rate of the eligibility trace *lambda*, with p < 0.01. We also observed a significant positive correlation between first level *beta* and the free parameter *w*¸ which governed the relative contributions of each type of model, with p < 0.01. The second level *beta* and *p*, the weighting coefficient of the indicator function that detected whether the same first level choice was made on the previous trial were significantly positively correlated as well, with p < 0.01. Finally, the negative correlation between *lambda* and *w* was observed to be significant, with p < 0.01. For a full summary of the results, see Table 4.3

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Eta 1st** | **Eta 2nd** | **Beta 1st** | **Beta 2nd** | **Lambda** | **W** | **P** | **RT 1** | **RT 2** |
| **Eta 1st** |  | 0.419 | 0.084 | -0.281 | -0.488 | 0.424 | -0.153 | -0.167 | -0.303 |
| **Eta 2nd** | 0.106 |  | 0.05 | -0.26 | -0.029 | 0.267 | -0.003 | 0.472 | 0.082 |
| **Beta 1st** | 0.758 | 0.854 |  | -0.285 | -0.703 | 0.723 | -0.274 | 0.224 | -0.12 |
| **Beta 2nd** | 0.291 | 0.33 | 0.284 |  | 0.504 | -0.413 | 0.653 | 0.203 | 0.357 |
| **Lambda** | 0.055 | 0.915 | **0.002** | 0.046 |  | -0.746 | 0.419 | 0.164 | 0.357 |
| **W** | 0.102 | 0.318 | **0.002** | 0.111 | **0.001** |  | -0.241 | 0.242 | -0.045 |
| **P** | 0.571 | 0.991 | 0.305 | **0.006** | 0.106 | 0.368 |  | 0.446 | 0.448 |
| **RT 1** | 0.536 | 0.065 | 0.405 | 0.45 | 0.544 | 0.367 | 0.083 |  | 0.403 |
| **RT 2** | 0.254 | 0.763 | 0.659 | 0.175 | 0.175 | 0.869 | 0.082 | 0.122 |  |

*Table 4.2: Correlations between Model Parameters and Reaction Time*

Correlation analyses were conducted between all possible pairwise combinations of the parameters used to define the model, as well as first and second stage average reaction times. The top diagonal lists the calculated correlation coefficients, while the bottom triangle includes the corresponding p-values. Correlations that were significant (p < 0.01) have been bolded and highlighted in blue.

# Discussion

## 5.1 Pairwise Analyses

There were a number of significant correlations found between the parameters in the model. We will address each in turn, beginning with those involving the first level inverse temperature parameter *beta*. *Beta* governs how probabilistic people’s decisions were. A subject with *beta* equal to zero would be uniformly random in their decision making, while a subject with a higher *beta* would be much more deterministic

This can help us interpret the negative correlation between *beta* and *lambda*. *Lambda* directly controls the magnitude of the secondary updates of the first level values, to reflect the latest changes in the second level after second level decisions have been made. For subjects who were more random in their first level decisions (smaller *beta*), these new updates were critical. To continue to maximize reward while making random decisions, subjects must continue to learn. These strong updates may be indicative of a conflict between random decision making and the need to update stored first level values. The correlation is reflective of the structure of the code as well. The *lambda* parameter is only used to update values in model free agents at the first stage, so it makes sense that two parameters associated only with the first stage were correlated. The same approach can be used to understand the negative correlation between *w* and *lambda.* As described above, *lambda* is only used in model free agents, so it makes sense that as people become more model based, there’s less dependence on lambda as an update mechanism.

Similar logic can be used to understand the positive correlation between *w* and *beta* at the first level. As subjects became more model free in their decision making (decreased *w*), they became less deterministic. This fits with the theory of reinforcement learning. Exploration is critical for model free learning. The reverse is true as well. Subjects who were model based in their decision making would have a better understanding of their environment, and therefore it may be preferable for them to focus on exploitation instead of exploration. There may be an effect due to the structure of the code, similar to the one described above, as well. Model free and model based agents differ only in the first stage, so it is not surprising that there was a correlation between the first level inverse temperature parameter and *w*.

Finally, we will consider the correlation between the second level *beta* values and the *p* coefficients. The *p* coefficient was representative of how much additional weight was given to first stage choices where the same choice was made in the previous trial. Subjects who were less deterministic in their second stage choices tended to give more weight to repeated choices. This increased reliance on *p* may serve to offset some of the randomness associated with these subject’s choices, ensuring subjects did not stray too far from the optimal reward choices.

## 5.2 Correlations with Reaction Time

There are a number of possible explanations why a stronger correlation between the model’s parameters and either the first or the second stage reaction time was not found. First and foremost, we only considered each subject’s data and associated models based upon the study as a whole. The models were constructed based on 201 consecutive trials, which provides insight only into the subject’s run as a whole. However, one might expect differences in the models at different points in the study For example, there might be learning period at the beginning of the study for each subject, where their responses were more model free. Similarly, if at a particular point in the study a subject identified a particular second level choice to be particularly rewarding, their responses might be more model based as they attempted to seek out that trial. By constructing the models based on the subjects’ data as a whole, and using average reaction time, it is possible that more nuanced relationships contained in the data were missed.

The weak correlation between the model’s parameters and reaction time may also point towards the pre-caching updated values of state-action pairs. It’s possible that subjects computed an updated estimate of the value for a the most recently visited state and corresponding action immediately after the action was made, instead of the next time the value was needed. By precompliling the values in this fashion, the models could make model based decisions as quickly as model free ones.

The Otto study, by demonstrating an increased preference for model free learning on trials where processing was interfered with *at the time of decision making*, indicates that values are updated as needed, instead of immediately after decision making. This seems to contradict the theory of precaching, where values are updated and stored for later after decisions are made. However, the Stroop task used in the Otto study was much more computationally demanding than Daw’s decision tree. Furthermore, subjects were instructed to focus on the Stroop task, leaving Daw’s decision tree to be of secondary importance. It is possible, especially while giving full attention to the Daw’s computationally simple decision tree, that the subjects were able to update the expected values of the previous state while simultaneously making a decision in the next state. In this manner, the updated values would be cached for future use while the agent at the same time as the agent was making its decision, minimizing differences in reaction time between model free and model based processes.

There are more nuanced factors that may have contributed to difficulty in predicting reaction time as well. The task used, while elegant in its ability to disentangle the contributions of model free and model based agents, is itself not very complex. The simplicity of the task may allow the subjects to create the models quickly, with minimal cognitive effort. This would result in minimal differences between model free and model based response times.

The reaction times may also be moderated by other effects that we did not account for. One likely candidate is the difficulty of discrimination between choices at the next stage. In either case, the model updates its estimated values of the previous stage based on the expected value for the next stage. The values of the first stage are updated according to the models estimate of the choices it has in the second stage, and the values of the second stage are similarly updated based on the rewards it expects after each choice. If one choice at either level is obviously the better of the options available, it should be easy for the agent to make a decision, resulting in a quick reaction time. However, if the values for each of the choices are similar to each other, it may be more difficult for the agent to choose between them. In this case, one would expect to see a slower reaction time.

Finally, there remain two relatively simple explanations for the lack of observed correlation. One of the simplest explanations could stem from the small sample size. After removing one subject as described above, only sixteen subjects remain. If the effect size was subtle, or moderated by other aspects of the study, it could be difficult to detect. Another possible explanation could be effects of subject fatigue throughout the study. Subjects were required to make a total of 402 decisions throughout the course of the study, excluding the training tasks. The study itself averaged over twenty minutes. It can be difficult to maintain attention on a repetitive task for long durations of time, and only minimal analyses were conducted during or after the study to identify potential trials during which the subject was inattentive.

# Suggestions for Further Study

There are a number of potential avenues for further research using the experimental paradigm Daw has developed (or slightly modified versions.) To address potential issues raised in the discussion, the study should be expanded to include more participants. An algorithm, operating both during and after the study has been completed, should be put in place to detect subjects who may be suffering from the fatigue effect. This algorithm could rely on on-line updates of the *beta*  parameters; if subjects became too random in their decision making, the test could request they re-focus on the study. Alternatively, modern cycle detection analysis (see Shmueli, 1982) could be used to detect repetitive patterns in subject responses.

Additionally, further analysis of the current data should be conducted to determine correlations between reaction time and model parameters on more finite scale. Multiple models could be constructed for each subject, using blocks of twenty five or more trials per model. This would allow a more nuanced investigation of the underlying relationships between the parameters and reaction times. The use of the average reaction time of each subject, even after extreme trials are removed, may not detect some of the more subtle relationships hidden in the data.

It would also be of interest to vary the allowed response time (being sure to instruct subjects of the change) or to set a minimum response time for subjects. A follow up study could place subjects into one of three groups: one group where subjects were forced to respond under ~750 milliseconds, one group where subjects were required to give a response between ~750 and ~1750 milliseconds, and one where subjects were forced to wait a minimum of ~1750 milliseconds before responding. The study could then examine whether any differences in the relative contributions of model free and model based agents between groups

More neurobiological approaches should be considered as well. Recent literature (Hampton, Bossaerts, and Doherty, 2006) points to the ventromedial prefrontal cortex (vmPFC) as a potential substrate for model based decision making. The use of transcranial magnetic stimulation (TMS) to selectively inactive the vmPFC, coupled with Daw’s decision making paradigm, would provide a way to quantitatively evaluate this hypothesis.

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**Appendix A**

*Supplementary Tables*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Estimate** | **Std. Error** | | **t value** | **Pr(>|t|)** |
| **(Intercept)** | -0.2343974 | | 0.670585 | -0.35 | 0.732 |
| **RT1** | 0.0008934 | 0.0009589 | | 0.932 | 0.367 |

*Table A.1: Correlation between w and RT1*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Estimate** | **Std. Error** | **t value** | **Pr(>|t|)** | **Signif** |
| **(Intercept)** | 524.2 | 120.567 | 4.348 | 0.00245 | \*\* |
| **eta1** | -92.354 | 75.636 | -1.221 | 0.25684 |  |
| **eta2** | 146.173 | 68.493 | 2.134 | 0.06537 | x |
| **beta1** | 5.702 | 7.617 | 0.749 | 0.47551 |  |
| **beta2** | 3.337 | 11.366 | 0.294 | 0.77656 |  |
| **lambda** | 75.921 | 114.729 | 0.662 | 0.52673 |  |
| **w** | 107.374 | 95.883 | 1.12 | 0.29527 |  |
| **p** | 208.94 | 143.906 | 1.452 | 0.18458 |  |

*Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘x’ 0.1 ‘ ’ 1*

*Table A.2: Correlation between Model Parameters and RT1*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Estimate** | **Std. Error** | **t value** | **Pr(>|t|)** | **Signif.** |
| **(Intercept)** | 503.644 | 205.893 | 2.446 | 0.0582 | x |
| **w** | 226.497 | 488.656 | 0.464 | 0.6625 |  |
| **eta1** | -74.744 | 156.736 | -0.477 | 0.6536 |  |
| **eta2** | 156.167 | 124.282 | 1.257 | 0.2644 |  |
| **beta1** | 0.775 | 11.561 | 0.067 | 0.9492 |  |
| **beta2** | 4.752 | 16.261 | 0.292 | 0.7819 |  |
| **lambda** | 99.753 | 171.241 | 0.583 | 0.5855 |  |
| **p** | 286.181 | 230.031 | 1.244 | 0.2686 |  |
| **w:eta1** | 32.475 | 397.671 | 0.082 | 0.9381 |  |
| **w:lambda** | -153.33 | 602.135 | -0.255 | 0.8091 |  |
| **w:p** | -473.451 | 823.391 | -0.575 | 0.5902 |  |

*Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘x’ 0.1 ‘ ’ 1*

*Table A.3: Correlation between Model Parameters and RT1,*

*with the inclusion of possible interactions with w*

**Appendix B**

*Hybrid Model Construction*

function LL = rllik\_hybrid(eta1, eta2, beta1, beta2, lambda, w , p, state,choice1,choice2,money)

%

% Adapted from code provided at seminar by Professor Yael Niv

%

% LL = rllik(eta,beta,lambda,state,choice1,choice2,money)

%

% output:

% LL - the log likelihood of the data

% input:

% eta1/2 - learning rate, first level/second level

% beta1/2 - softmax inverse temperature, first level/second level

% lambda - eligibility trace decay rat (set to 0 to get TD(0) without eligibility traces)

% state - 1 is the top level, 2 and 3 are the bottom level

% choice1 - the choice at the top level -- 1 or 2 (0 for missed trials)

% choice2 - the choice at the bottom level -- 1 or 2 (0 for missed trials)

% money - amount won (1 or 0)

% w - percent model free/model based

% p - weighting for consecutive identical first stage choices

NStates = 3;

NActions = 2;

NTrials = length(choice1);

Qfree = zeros(NStates,NActions); % initialize Qfree values to 0

Qbased = zeros(NStates,NActions); % initialize Qbased values to 0

Qhybrid = zeros(NStates,NActions);

LL = 0 ; % initialize log likelihood

prev = 0;

% main loop

for t = 1:NTrials

repa = zeros(1,NActions);

% stop if trial was missed

if (choice1(t) == 0 || choice2(t) == 0)

continue

end

other = 0;

if choice1(t) == 1

other = 2;

end

if choice1(t) == 2

other =1;

end

if other == 0

ff('bad first level choice, choice = %d\n', choice1(t))

continue

end

%assign p

if t>1

if choice1(t-1) ~= 0

repa(choice1(t-1)) = 1;

end

end

% first level choice likelihood

LL = LL + beta1\*(Qhybrid(1,choice1(t)) + p \* repa(choice1(t))) - logsumexp(beta1\*(Qhybrid(1,:) + p \* repa));

% second level choice likelihood

LL = LL + beta2\*Qhybrid(state(t),choice2(t)) - logsumexp(beta2\*Qhybrid(state(t),:));

%first level model free

PE = Qfree(state(t),choice2(t)) - Qfree(1,choice1(t)); %SARSA

Qfree(1,choice1(t)) = Qfree(1,choice1(t)) + eta1\*PE;

% second level model free

PE = money(t) - Qfree(state(t),choice2(t));

%Update second level model free

Qfree(state(t),choice2(t)) = Qfree(state(t),choice2(t)) + eta2\*PE;

%Update 1st level again model free

Qfree(1,choice1(t)) = Qfree(1,choice1(t)) + eta1\*PE\*lambda;

% learning at second level model based

PE = money(t) - Qbased(state(t),choice2(t)) ;

%Update second level model based

Qbased(state(t),choice2(t)) = Qbased(state(t),choice2(t)) + eta2 \* PE;

%initialize count parameters

left = 0;

right = 0;

%determine which transition occurs more frequently, and thus receives

%the higher weighting

for i = 1:t

if choice1(i) == 1 && state(i) == 2

left = left + 1;

end

if choice1(i) == 2 && state(i) == 3

left = left + 1;

end

if choice1(i) == 1 && state(i) == 3

right = right + 1;

end

if choice1(i) == 2 && state(i) == 2

right = right + 1;

end

end

%assign weighting based on most frequent choice

if left > right

weight2 = 0.7;

weight3 =0.3;

else

weight2 = 0.3;

weight3 = 0.7;

end

%first level model based

%Bellman Equation

Qbased(1,1) = weight2 \* max(Qbased(2,:)) + weight3 \* max(Qbased(3,:));

Qbased(1,2) = weight3 \* max(Qbased(2,:)) + weight2 \* max(Qbased(3,:));

% hybrid update

Qhybrid = w \* Qbased + (1-w) \* Qfree;

end

% we are minimizing this function, so use minus LL

LL = -LL;

**Appendix C**

*Hybrid Model Driver*

%

% Adapted from code provided at seminar by Professor Yael Niv

%

%clear all; clc;

Subjects = [17];

Nsubjects = 17;

C1 = []; C2 = []; R = []; S = []; subj = []; react1 = []; react2 = [];

% loading the subjects’ behavioral data

SubjFile = dir('dawdatatrans.mat');

% parsing the subjects’ behavioral data

for s = 1:Nsubjects

offset = ((s-1) \* 201) + 1;

endoffset = offset + 200;

C1 = [C1; ch1(offset:endoffset)]; % the choices at level 1

C2 = [C2; ch2(offset:endoffset)]; % the choices at level 2

R = [R; mn(offset:endoffset)]; % the rewards

S = [S; st(offset:endoffset)]; % the states at level 2

react1 = [react1; rt1(offset:endoffset)]; %first stage RT

react2 = [react2; rt2(offset:endoffset)]; %second stage RT

end

[Nsubjects,Ntrials] = size(S);

optset = optimset('algorithm', 'sqp', 'Display', 'off');

Fit = {};

clear Eta1 Eta2 Beta1 Beta2 Lambda w p

for iter = 1:10; % run 10 times from random initial conditions, to get best fit

for i = 1:Nsubjects;

ff('%d...',i)

LB = [1e-6 1e-6 1e-6 1e-6 1e-6 1e-6 -10];

UB = [1-(1e-6) 1-(1e-6) 20 20 1-(1e-6) 1-(1e-6) 10];

init = rand(1,length(LB)).\*(UB-LB)+LB; % random initialization within the bounds

% finding the minimum of the function rllik

[res lik] = fmincon(@(x) rllik\_hybrid(x(1),x(2),x(3),x(4),x(5), x(6), x(7), S(i,:),C1(i,:),C2(i,:),R(i,:)),init,[],[],[],[],LB,UB,[],...

optset);

% gathering results

Eta1(i) = res(1);

Eta2(i) = res(2);

Beta1(i) = res(3);

Beta2(i) = res(4);

Lambda(i) = res(5);

w(i) = res(6);

p(i) = res(7);

Lik(i) = lik;

end

ff('\n')

Fit{iter} = [[1:Nsubjects]' Eta1' Eta2' Beta1' Beta2' Lambda' w' p' Lik'];

L(:,iter) = Lik'; % Check this to see the likelihoods from the different runs (to check how stable the fits were to different starting points)

end

% find the best fit of all 10 runs

clear BestFit

[a,b] = min(L');

for i = 1:Nsubjects

BestFit(i,:) = Fit{b(i)}(i,:);

end

% the results

ff('Sub\t eta1\t eta2\t beta1\t beta2\t lambda\t w\t p\t LL\n')

ff('%d\t %3.3f\t %3.3f\t %3.3f\t %3.3f\t %3.3f\t %3.3f\t%3.3f\t %3.3f\t\n',BestFit')